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## SURFACING OF A HEAVY SPHERE IN VIBRATING SAND

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This article reports the results of an experimental study of the motion of a sphere in a vibrating free-flowing medium. Analysis of the results made it possible to establish that the mean upward velocity of the sphere is linearly dependent on its diameter.

The effects of the motion of a uniform, free-flowing vibrating medium in an oscillating field of accelerations comparable to (vibro-fluidization regime) or greater than (vibro-boiling regime) gravitational acceleration with regard to amplitude have been studied in fairly great detail and are being put to use in industry [1]. Several experimental [2] and theoretical [3] studies have examined the surfacing of heavy bodies and immersion of light bodies in a vibrating liquid. Numerical methods were used in conjunction with a two-dimensional model in [4] to study the laws governing the vibrational separation (segregation) of a uniform mixture of particles of two very different diameters. As far as we know, there have not been any experimental studies of the vertical motion of a single sphere in a horizontal layer of a vibrating free-flowing medium. However, this problem is of interest both for technical applications and for geophysics, in connection with the need to investigate the anomalous "surfacing" effects noted in the literature in earthquakes involving the movement of large masses of soil and boulders under fine-grained sedimentary rock [5].

The experiments were conducted on a vibration unit which provided for motion of the free-flowing medium in a variable gravitational field created by vertical oscillations of the vibrator. The unit was a cylindrical container 50 cm in diameter and 20 cm high. Installed in the bottom part of the container was a plane coaxial vibrator 20 cm in diameter (Fig. 1). The vibrator ensured vibration of a layer of dry sand up to 15 cm thick with accelerations up to  $25 \text{ m/sec}^2$  and a constant vibration amplitude of 0.1 cm. The frequency of vibration ranged from 1 to 25 Hz. The test sphere was placed at the bottom of the sand layer. After the vibrator was turned on and had operated for a certain period of time at a certain frequency, the sphere ended up on the surface of the sand. The thickness of the sand layer was changed with the range from 4 to 10 cm and was measured in each test both before the vibrator was turned on and after the sphere had risen to the surface. We used spheres ranging in diameter  $D$  from 4 to 45 mm. They were made of wood, cork, clay, paraffin, glass, polystyrene, iron, aluminum, and lead.

In the main test series, we recorded the average velocity of the sphere  $V$  as the ratio of the thickness of the sand layer to the average time of movement of the sphere from the bottom to the surface measured with a stopwatch. We also studied the law of motion of the sphere with the use of a system (Fig. 1) which included a thin inextensible filament, a lightweight dial gauge, and an extensible elastic filament (to raise the pointer). The use of a system with an elastic filament made it possible to determine the minimum frequency at which the sphere begins to rise and to fix the moment of the beginning of motion and the change in velocity over the initial section of the sphere's path. More than 200 tests were conducted with spheres of different diameters. The error of the determination of velocity in a given measurement was no greater than 5%. The slightly tensioned rubber filament did not significantly distort the measurement results.

The completed tests allowed us to find the threshold frequency at which the sphere surfaces. This value was found to be  $f_{\min} = 20 \text{ Hz}$ . The tests also made it possible to fix the initial velocity of the sphere ( $V_{\min} = 0.1 \text{ mm/sec}$ ) and to study the character of the

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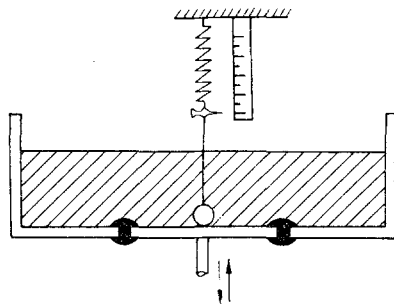


Fig. 1

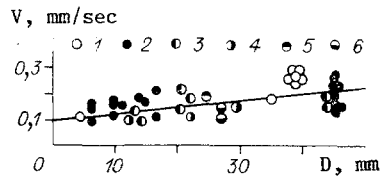


Fig. 2

change in the mean upward velocity of different-diameter spheres made of different materials within the range of vibrator frequencies from 20 to 24 Hz. This range corresponds to mean values of acceleration of 8-11.5 m/sec<sup>2</sup>.

Analysis of the measurements of the mean velocity of the spheres with allowance for their trajectories in the sand allowed us to establish the following dependence of sphere velocity  $V$  (mm/sec) on sphere diameter  $D$  (mm):  $V = \alpha + \beta D$ . Here,  $\alpha = 0.1 \pm 0.02$ ,  $\beta = 2.5 \cdot 10^{-3}$  are numerical coefficients.

The dependence of velocity on diameter, illustrated in Fig. 2, is valid within the vibrator-frequency range from 20 to 24 Hz and turns out to be nearly insensitive to changes in the density of the material of the sphere. The density of the material ranged from 0.3 to 11.3 g/cm<sup>3</sup> in the different tests (the numbers of the points correspond to the following: 1 - lead; 2 - iron; 3 - glass; 4 - polystyrene; 5 - paraffin; 6 - cork).

An additional series of tests revealed that when the unit is operated in the vibro-boiling regime ( $f \geq 25$  Hz), mean sphere velocity increases by nearly an order of magnitude. Although the overall character of the dependence of velocity on diameter remains the same as before, there is an appreciable increase in the scatter and the measurement error. It was established with the aid of markings made on the spheres that they retain their original spatial orientation (do not rotate) during their ascent.

A phenomenological model of the displacement of a sphere in vibrating sand can be constructed by using the "ratchet" principle. The sphere is displaced upward under the influence of a vibrational force, thus displacing a certain small volume of sand in a vibrationally fluidized state. Little or no downward movement of the sphere occurs in the adjacent half-cycle, since the volume momentarily freed of sand under the sphere is filled with other sand particles while the sphere is rising.

Whereas a vibration velocity on the order of 1 m/sec is required for the surfacing of a heavy body in liquid [3], the same spherical body surfaces in vibrating sand at velocities of about 10 cm/sec with mean accelerations on the order of 8 m/sec<sup>2</sup>. Such values of velocity and acceleration are typical of strong earthquakes.

It follows from this that the mean upward velocity of a mass of earth 1 m in diameter during strong earthquakes may reach from 1 to 10 cm/sec - in accordance with the established empirical relations. Considering that the mean duration of an earthquake of a magnitude greater than 7.5 (on the Richter scale) is approximately 100 sec, we calculate that the mass would be displaced upward in loose soil by 1-10 m during one seismic event.

Tens or even hundreds of earthquakes may occur in a given region during a geological period, these quakes leading to the upward displacement of large masses of earth by tens of meters. Anomalies in the granulometric composition of soil in which large masses are located above finely dispersed material have been noted in the geological literature on seismically active regions [5, p. 146] and are given a plausible explanation within the framework of the above representations on the surfacing of bodies in vibrating sand.

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MOTION OF A CHAIN OF BUBBLES IN A VERTICAL CHANNEL WITH  
A VISCOUS FLUID

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Regular chains of nearly identically shaped bubbles moving one after the other are seen in bubble towers and in the uniform passage of gas bubbles through a tube. The upward velocity of the bubbles in these cases will obviously differ from the velocity of a single bubble of the same size, while the pattern of flow will depend to a significant extent on the distance between the bubbles. There are still serious obstacles to conducting theoretical and experimental studies of such bubble motion. The study [1] – where Nakorjakov et al. obtained the longitudinal components of the velocity vector across a tube behind a bubble and measured the friction on the tube wall – might be the only investigation to give some quantitative picture of the hydrodynamics of the process.

In the present study, we propose an algorithm for numerically solving the problem of the steady motion of a chain of bubbles in a viscous fluid within a vertical tube under the influence of buoyancy. Results are obtained for the case when the tube wall has almost no effect on the rise of the bubbles and the case when this effect is decisive.

1. Formulation of the Problem. In the coordinate system connected with the center of mass of the chain of bubbles, the tube moves downward at a constant velocity  $u$  equal to the upward velocity of each bubble and the fluid flows past the chain. In this case, the motion of the entire chain can be described by examining the flow past a single bubble. Since the problem is periodic, the flow pattern will be the same over the distances  $L$  up and down from the center of mass of the bubble; the period will be equal to  $2L$ .

We introduce a spherical coordinate system  $(r, \theta, \varphi)$  whose origin  $O$  coincides with the center of mass of the bubble (Fig. 1). Let  $r = R(\theta)$  ( $\theta \in [0, \pi]$ ) be the equation of the surface of the bubble and  $R_k$  be the radius of the tube. With allowance for the axial symmetry of the problem, the Navier-Stokes equations of a viscous incompressible fluid in the variables  $\omega$  (curl) and  $\psi$  (stream function) have the same form as in [2, 3].

The boundary conditions differ from those described in [3] only in the sections  $\Gamma_1$  ( $r = L/\cos\theta$ ,  $\theta \in [0, \theta^*]$ ) and  $\Gamma_2$  ( $r = -L/\cos\theta$ ,  $\theta \in [\pi - \theta^*, \pi]$ ,  $\tan\theta^* = R_k/L$ ), where we assign periodicity conditions reflecting the fact that the values of the velocity functions and their derivatives over normals to the boundaries  $\Gamma_1$  and  $\Gamma_2$  coincide. It follows from [4] that, in terms of the curl  $\omega$  and stream function  $\psi$ , they have the following form (where the  $z$  axis is directed along the tube)

$$\begin{aligned} \psi|_{\Gamma_1} &= \psi|_{\Gamma_2}, \quad \partial\psi/\partial z|_{\Gamma_1} = \partial\psi/\partial z|_{\Gamma_2}, \\ \omega|_{\Gamma_1} &= \omega|_{\Gamma_2}, \quad \partial\omega/\partial z|_{\Gamma_1} = \partial\omega/\partial z|_{\Gamma_2}. \end{aligned} \tag{1.1}$$